NONMAGNETIC CONTAINMENT OF A DENSE PLASMA

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A study is made of the possibility of confining a thermonuclear plasma with temperature $T \sim 10^4$ eV and density $n \sim 10^{18}$ cm^{-3}, not by magnetic field pressure, but by hard walls of a chamber (nonmagnetic containment). This method of plasma containment has some specific features: the occurrence of plasma flow, formation of a dense layer at the wall, increased importance of radiative losses from the plasma, and more. A numerical solution of the plasma-transport equations is used to investigate the influence of these features on the energy lifetime of the plasma. The results indicate that the additional energy losses by the plasma are not catastrophically large, and, in principle, nonmagnetic containment of a dense plasma is possible.

1. Description of the Problem

In traditional systems of containment of a plasma with density $n \sim 10^{13}$-10^{14} cm^{-3} the main problem is to guarantee reliable separation of the plasma from the chamber walls. This is possible only under conditions when the magnetic pressure $H^2/8\pi$ exceeds the gas-kinetic pressure 2nT of the plasma (magnetic containment). On the transition to a plasma with density $n \sim 10^{17}$-10^{18} cm^{-3} and temperature $T \sim 10^4$ eV, magnetic containment requires the use of magnetic fields in the megagauss range, and these are hard to produce. It is therefore of interest to consider the possibility of confining a dense plasma by the walls of a chamber ("nonmagnetic" containment). With this method of containment, the magnetic field is required only to reduce the transverse heat conduction, which can already be achieved with very moderate magnetic fields (such that $\beta = 16\,\pi nT/H^2 \gg 1$). The possibility of nonmagnetic containment was already noted by Sakharov [1], and since then has been frequently mentioned by different authors.

The characteristic differences of nonmagnetic containment of a plasma with $\beta \gg 1$ from the case of small β are most clearly manifested in a problem with initial conditions. Suppose at the initial time a homogeneous cold plasma fills the interval between two flat parallel walls (for simplicity, we consider the case of planar geometry). Since in practically interesting cases the heating time* is appreciably longer than the inertial time, the total pressure $2nT + H^2/8\pi$ in the plasma will also be homogeneous over the section after the heat source has been switched on. If $\beta \ll 1$, then mechanical equilibrium in the system is guaranteed by the pressure of the magnetic field, and one can assume that the heating takes place at fixed density of the plasma (it is assumed that the heating time is much shorter than the plasma diffusion time at right angles to the magnetic field). But in the case of a plasma with $\beta \gg 1$ the condition nT = const must be satisfied, i.e., $n \sim 1/T$. Therefore, as it is heated the plasma will flow out from the relatively hot central regions to the cold walls, where a region with very high density develops.

The presence of this cold wall layer can, in principle, lead to a sharp increase in the role of bremsstrahlung, which can leave the region of the plasma freely. The point is that the emission power from unit volume of the plasma, Q_{rad}, which is proportional to $n^2\sqrt{T}$,

*The heat source can, in particular, be a relativistic electron beam.

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increases as $T^{-3/2}$ with decreasing temperature subject to the condition nT = const. In addition, flow of the plasma gives an additional flux of heat to the walls (in excess of the diffusion flux). The aim of this paper is to establish how these features of nonmagnetic containment influence the dynamic heating of a dense plasma and its energy lifetime. Such an investigation is important, since for a thermonuclear plasma with temperature $T \sim 10^4$ eV the time of radiative cooling $\tau_{rad} = 3nT/Q_{rad}$ even for a D–T mixture is only an order of magnitude greater than the necessary time of plasma containment that follows from the Lawson criterion: $\tau_\Lambda(\text{sec}) = 10^{14}/n(\text{cm}^{-3})$. This increase in Q_{rad} could mean that nonmagnetic plasma containment cannot, in principle, be used.

The problem of nonmagnetic containment of a plasma is made very much more complicated by the need to take into account various effects that occur directly next to the wall such as the incomplete ionization of the plasma and the recombination radiation, the violation of the condition that the radiation should not be trapped, the nonideal behavior of the plasma and the related changes in its transport coefficients, destruction of the wall by the heat flux and fast particles of the plasma, and penetration into the plasma of heavy impurities from the wall.

In this paper, we consider only the behavior of the plasma in the regions with relatively high temperature ($T \geq 10$ eV). The presence of the wall is taken into account in the form of some simple boundary conditions imposed on the parameters of the plasma at the point where its temperature is still sufficiently high for one to be able to ignore the wall effects listed above (in the cases discussed below, the temperature of this "effective" wall is chosen in the range 3 to 100 eV).

Analysis of the questions relating to the increased radiative loss and convective heat transport to the cold walls in the framework of this simplified model shows that for reasonable parameters of the plasma its lifetime satisfies the Lawson criterion with some margin.

However, it must be borne in mind that this result has the meaning of only a necessary condition for the possibility of nonmagnetic containment, since the role of the processes that take place at the wall itself (in the region $T \leq 3$ eV) is not entirely clear. However, there are some indications that these processes are not too important: Variation of the temperature of the effective wall has little influence on the characteristics of the hot plasma (for details see §§3 and 4).

In the plasma one can have the excitation of various microinstabilities (the system considered here is stable against the most dangerous magnetohydrodynamic instabilities), and therefore, besides the classical transport coefficients, we also consider the case of anomalous (Bohm) thermal conductivity of the plasma. The results obtained give correspondingly approximate values of the upper and lower limits of the energy lifetime of the plasma.

2. Statement of the Problem

We consider a layer of plasma bounded by fixed perfectly conducting planes $x = \pm L$ whose temperature is assumed constant. The magnetic field is parallel to the walls. The equations of plasma transport at right angles to the magnetic field can be written in the form (notation as in [2])

$$
\begin{gathered}
Mn\frac{dv}{dt} = -\frac{\partial}{\partial x}\left(2nT + \frac{H^2}{8\pi}\right), \qquad \frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0 \\
\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(vH) = \frac{c^2}{4\pi}\frac{\partial}{\partial x}\left(\frac{1}{\sigma_\perp}\frac{\partial H}{\partial x}\right) + \frac{c}{e}\frac{\partial}{\partial x}\left(\frac{\beta_\Lambda{}^{uT}}{n}\frac{\partial T}{\partial x}\right) \\
3\frac{\partial}{\partial t}(nT) + 3\frac{\partial}{\partial x}(nTv) + 2nT\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(\varkappa_\perp\frac{\partial T}{\partial x} + \frac{c\beta_\Lambda{}^{uT}T}{4\pi ne}\frac{\partial H}{\partial x}\right) + \\
+\frac{c\beta_\Lambda{}^{uT}}{4\pi ne}\frac{\partial H}{\partial x}\frac{\partial T}{\partial x} + \frac{c^2}{16\pi^2\sigma_\perp}\left(\frac{\partial H}{\partial x}\right)^2 + Q_i - Q_{\text{rad}}
\end{gathered}
\tag{2.1}
$$

Here v is the mass velocity of the ions directed along the x axis perpendicular to the walls; $\varkappa_\perp$ and $\sigma_\perp$ are the thermal conductivity and the conductivity of the plasma at right angles to the magnetic field; $\beta_\Lambda{}^{uT}$ is the coefficient that determines the component of the thermal force at right angles to ∇T and **H**:

$$Q_{\text{rad}} = 10^{-13}(n\text{cm}^3)^2\sqrt{T(\text{eV})^{-1}}\ \text{eV}\cdot\text{cm}^{-3}\cdot\text{sec}^{-1}$$

Q_{rad} is the power of the bremsstrahlung leaving the plasma (see, for example, [3])

$$Q_t = W\{\exp[1-(x/L)^2]-1\}\,t\exp(-t/\Delta t)\,(e-1)^{-1}(\Delta t)^{-2}$$

Q_t is the volume power of the external heat sources (this dependence can imitate, for example, the heating of the plasma by a relativistic electron beam).

Thermonuclear heat sources are not included in the problem, since in our opinion it is at present more important to calculate model experiments in which the maximal temperature of the plasma is several times lower than the thermonuclear.

The temperatures and densities of the electrons and ions can be assumed equal, the radiation regarded as untrapped, and viscous effects regarded as unimportant. As we have already noted, inertial effects are small, but the inertial terms are retained in the equations of motion for convenience in numerical integration.

The system (2.1) was solved numerically on the interval $0 \le x \le L$ with the initial and boundary conditions

$$\begin{aligned}
&t=0,\quad n=n_0,\quad H=H_0,\quad T=T_x,\quad v=0\\
&x=0,\quad \frac{\partial T}{\partial x}=\frac{\partial H}{\partial x}=0,\quad v=0\\
&x=L,\quad T=T_x,\ v=0\\
&\frac{c}{4\pi\sigma_\perp}\frac{\partial H}{\partial x}+\frac{\beta_\Lambda^{uT}}{ne}\frac{\partial T}{\partial x}=0
\end{aligned}$$

The last relation describes the boundary condition on a perfectly conducting wall. This approximation is reasonable if the magnetic field penetrates during the time of the experiment into the wall through a distance that is short compared with the thickness of the "magnetic" wall layer $\Delta_H = (\partial \ln H/\partial x)^{-1}$:

$$\frac{c^2}{4\pi\sigma_\perp}t \ll \Delta_H{}^2$$

This condition for the values $t \sim 10^4$ sec and $\Delta_H \sim 6\cdot10^{-1}$ cm obtained below has the form

$$\sigma \gg 10^{16}\ \text{sec}^{-1}$$

and is satisfied for good conducting metals.

Some preliminary results of the calculations are contained in [4]. The problem of nonmagnetic containment of a dense plasma is also considered in [5-7]. In [5], a solution is found for the problem of stationary nonmagnetic containment by walls when the heat losses due to heat conduction and radiation are compensated by thermonuclear sources. It is found that the existence of stationary solutions with physical meaning depends strongly on the magnetic field profile. However, in [5] the magnetic field is not determined in a self-consistent manner, but specified arbitrarily.

In [6], a study is made of a stationary thermonuclear reactor with dense plasma in which the fuel (deuterium and tritium) continuously diffuses from the walls to the burning zone, while the reaction products (α particles) diffuse in the opposite direction. Compared with the present paper, the results of [6] correspond to a very different time scale. In this paper we are concerned with times of the order of the Lawson time, whereas the solution of [6] is established over a time that is much greater than the time of complete burning, which is two orders of magnitude longer than the Lawson time (for a D—T mixture). It should also be noted that in [6] the transport coefficients for a magnetized plasma are used, and this is not valid in the cold wall regions.

The dynamics of cooling of a dense plasma in contact with a cold wall is discussed in [7]. However, [7] deals with a plasma whose parameters are such that radiation is unimportant.

3. Classical Heat Conduction

In this case the characteristic time in the problem is the time of cooling of the hot plasma by ion heat conduction at right angles to the magnetic field (it is assumed that in the hot plasma the electrons and ions are magnetized and the plasma is a deuterium plasma):

$$\tau_k = L^2 e H_0 (m / M)^{1/2} [c T_0 \delta_0 (n_0, T_0, H_0)]^{-1}$$

The magnetization parameter $\delta_0 \equiv \nu_e/\omega_{He}$ is the ratio of the Coulomb frequency of scattering of electrons to their cyclotron frequency. The characteristic temperature T_0 is related to the heating power by $T_0 = W/3n_0$, so that the temperature in the center in the case of homogeneity of the plasma and in the absence of heat losses would increase to T_0. In reality, the maximal temperature in the center does not reach this value, since some of the energy is lost by heat conduction and radiation, and some is used to compress the colder layers of the plasma.

In Eqs. (2.1) and in the boundary conditions it is convenient to go over to dimensionless variables taking as the scales of x, n, H, T, t, and v the quantities L, n_0, H_0, τ_k, and τ_k/L, respectively:

$$\varepsilon n \frac{dv}{dt} = -\frac{\partial}{\partial x}\left(nT + \frac{H^2}{\beta_0}\right) \tag{3.1}$$

$$\frac{dn}{dt} + n\frac{\partial v}{\partial x} = 0 \tag{3.2}$$

$$\left(\frac{M}{m}\right)^{1/2} \delta_0 (n_0, T_0, H_0) \frac{d}{dt}\left(\frac{H}{n}\right) = \frac{4}{\beta_0 n}\frac{\partial}{\partial x}\left(\delta_0 \frac{H}{n}\frac{\partial H}{\partial x}\right) + \frac{1}{n}\frac{\partial}{\partial x}\left(\delta_1 \frac{\partial T}{\partial x}\right) \tag{3.3}$$

$$\left(\frac{M}{m}\right)^{1/2} \delta_0 (n_0, T_0, H_0)\left(n\frac{dT}{dt} - \frac{2}{3}T\frac{dn}{dt}\right) = \frac{\partial}{\partial x}\left(\frac{nT}{H}\delta_2\frac{\partial T}{\partial x}\right) + \frac{4}{3\beta_0}\frac{\partial}{\partial x}\left(\delta_1 T\frac{\partial H}{\partial x}\right) + \frac{4}{3\beta_0}\frac{\partial H}{\partial x}\left(\frac{4\delta_0}{\beta_0 n}H\frac{\partial H}{\partial x} + \delta_1\frac{\partial T}{\partial x}\right)$$
$$+ \left(\frac{M}{m}\right)^{1/2} \delta_0 (n_0, T_0, H_0)\left[\frac{t}{(\Delta t)^2}\exp(-t/\Delta t)\frac{\exp(1-x^2)-1}{e-1} - \alpha n^2 \sqrt{T}\frac{(T-\gamma)^2}{\gamma^2 + (T-\gamma)^2}\right] \tag{3.4}$$

$$v_{x=0} = v_{x=1} = 0, \quad T_{x=1} = \gamma; \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = \left.\frac{\partial H}{\partial x}\right|_{x=0} = 0$$
$$\left.\left(\frac{4\delta_0}{\beta_0}H\frac{\partial H}{\partial x} + \delta_1 n\frac{\partial T}{\partial x}\right)\right|_{x=1} = 0, \quad \beta_0 = 16\pi n_0 T_0 / H_0^2$$

Here in writing down the equations and the conditions for the dimensionless quantities we have retained the previous notation. The parameter $\varepsilon = ML^2/2T_0\tau_k^2$ characterizes the role of inertia of the plasma (in the cases of practical importance $\varepsilon \ll 1$), and α is equal to the ratio of the time τ_k to the time of radiative cooling of the hot plasma τ_{rad} ($\alpha = \tau_k/\tau_{rad}$, $\tau_{rad} = 3n_0T_0/Q_{rad}(n_0T_0)$). The quantity $\gamma \ll 1$ is the dimensionless temperature of the wall. The factor $(T-\gamma)^2/\gamma^2 + (T-\gamma)^2$ in the last term on the right-hand side of Eq. (3.4) takes into account the fact that the plasma cannot cool to a temperature below the wall temperature as a result of radiation. It follows from the numerical calculations that although the volume power of the radiation Qrad in the wall layer is high the total radiation $\int Q_{rad}dx$ from this layer is a small part of the radiation from the whole volume of the plasma. Therefore, the artificial elimination of the radiation from the wall has no practical influence on the results. The coefficients δ_1 and δ_2 are proportional, respectively, to the thermal force and the thermal conductivity. Their dependence on δ_0 was determined by means of interpolation formulas (see [2]):

$$\delta_1 = \frac{3\delta_0}{2+4\delta_0^2}, \quad \delta_2 = \delta_0\left(\frac{1}{0.6+\delta_0^2} + \frac{1}{3\cdot 10^{-2} + 37.5\delta_0^2}\right) \tag{3.5}$$

(the terms in δ_2 take into account the contributions of the electrons and ions, respectively, to the heat conduction). Note that the value of δ_0 at the wall may differ by several orders of magnitude from $\delta_0(n_0, T_0, H_0)$ and even become greater than unity (the plasma is demagnetized).

The results of the numerical calculations are illustrated by the figures and numerical data given in Table 1. The characteristic parameters of the problem were chosen as follows: $\varepsilon = 10^{-6}$, $\Delta t = 0.2$, $\alpha = 0.1$, $\delta_0(n_0, T_0, H_0) = 10^{-4}$, $\gamma = 10^{-3}$. (note that a further decrease of γ did not influence the nature of the processes in the region of hot plasma).

TABLE 1.

β_0	1	10	10^2	10^3
T_{max}	0.354	0.365	0.367	0.355
n_m	0.93	0.68	0.523	0.92
$\tau_{1/2}$	0.42	0.36	0.36	0.48

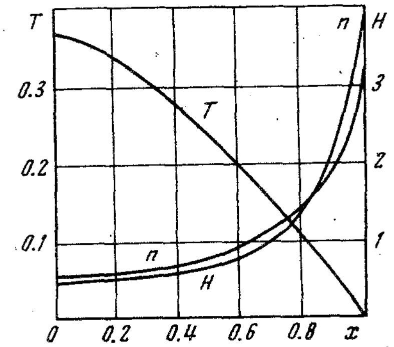

Fig. 1

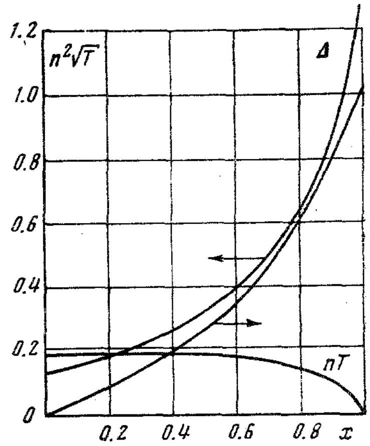

Fig. 2

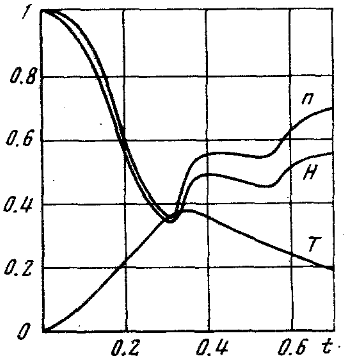

Fig. 3

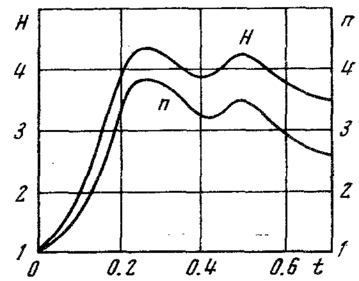

Fig. 4

Figure 1 shows the profiles of the density n, the temperature T, and the magnetic field H for $\beta_0 = 10^2$ at the time $t = t^*$ when the temperature in the center is maximal. As can be seen from Fig. 1, the formation of a wall layer in this case is not strongly expressed: $n_{max}/n_{min} = 6.3$. This is because the magnetic field is transported from the center with the plasma, and at the walls magnetic pressure plays the main role ($\beta|_{x=1} \ll 1$). In this case $H_{max}/H_{min} = 8.3$ (because of the presence of the thermal force this ratio is even greater than if the field were frozen into the plasma). The maximal temperature in the center is 0.37.

We also give the value of the "efficiency" η, which is equal to the ratio of the thermal energy stored in the system at the time $t = t^*$ to the total energy of the source. For the case considered, $\eta \simeq 0.28$. The rather small value of η and the low maximal temperature are explained by the fact that the characteristic heating time Δt is only half the plasma cooling time $\tau_{1/2}$, which is defined as the time during which the temperature in the center of the system falls to half the maximal value. In Table 1, we give the maximal temperature at the center of the system T_{max}, the density n_m in the center at the time $t = t^*$, and also the cooling times $\tau_{1/2}$ for different values of the parameter β_0.

Figure 2 shows the profiles of the plasma pressure nT, the radiation density $n^2\sqrt{T}$, and the emission $\Delta = \int_0^x n^2 \sqrt{T}\, dx \Big/ \int_0^1 n^2 \sqrt{T}\, dx$ for $\beta_0 = 10^2$ at the time t^*. Figures 3 and 4 are the density, the magnetic field, and the temperature as functions of the time, respectively, in the center of the system and at the wall. It can be seen from the graphs that when the heating ends a flow of plasma back to the center begins (the density in the center begins to increase). Figure 5 shows the magnetization profiles of the electrons and ions at t^*.

Let us give the values of some of the quantities for thermonuclear parameters of the plasma: $n_0 = 10^{18}$ cm^{-3}, $H_0 = 10^5$ G, $T_0 = 10^4$ eV. The transverse dimension L of the system is chosen in such a way that the cooling time $\tau_{1/2}$ is of the order of the Lawson time for

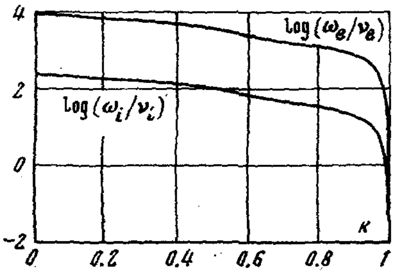

Fig. 5

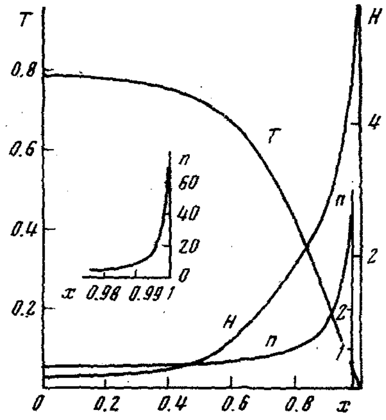

Fig. 6

the D–T mixture: $\tau_\Lambda = 10^{14}(\mathrm{n\ cm^3})^{-1}\mathrm{sec} = 10^{-4}$ sec. Then the transverse dimension of the system is $L \approx 2.2$ cm) the radiation losses are $\int Q_{\mathrm{rad}}dx = 6.4 \cdot 10^6\ \mathrm{J \cdot cm^{-2} \cdot sec^{-1}}$ ($t = t^*$); the heat flux to the wall is $q \approx 1.3 \cdot 10^7\ \mathrm{J \cdot cm^{-2} \cdot sec^{-1}}$ ($t = t^*$).

4. Bohm Heat Conduction

The influence of instabilities can be taken into account approximately in Eqs. (2.1) if ν_e is understood as some effective electron-scattering frequency, which can be much greater than the Coulomb one. In a given magnetic field, the thermal conductivity of the plasma is maximal for $\nu_e \approx \omega_{He}$ (this gives a thermal conductivity coefficient of the order of the Bohm coefficient). Therefore, in this section we shall assume that $\delta_0 = 1$.

Since the characteristic time is the Bohm time $\tau_B = L^2eH_0/cT_0$, it is convenient in this case to make the time and velocity dimensionless by dividing by τ_B and τ_B/L, respectively.

The remaining variables are made dimensionless in the same manner as in the classical case. The parameters ε and α are equal to

$$\varepsilon = ML^2 / 2T_0\tau_B^2, \quad \alpha = \tau_B / \tau_{\mathrm{rad}}$$

The values of ε, Δt, α, and γ in the calculations were the same as in the classical case.

The strength and sign of the thermal force are determined by the way in which the frequency of electron scattering on fluctuations depends on the electron velocity. Since, generally speaking, this dependence is unknown for the oscillations that lead to the Bohm transport coefficients, we have considered two cases: with thermal force determined by (3.5), and without thermal force, which corresponds to $\delta_1 = 0$. Comparison of the calculations in the two cases shows that the characteristics of the hot plasma then differs little, i.e., the results depend weakly on the actual form of the thermal force.

Figure 6 shows the n, T, and H profiles for $t = t^*$ with allowance for the thermal force. Figure 7 shows the same dependences in the absence of the thermal force. One can clearly note the formation of a thin cold layer at the wall with density approximately two orders of magnitude greater (~70 and ~270, respectively) than the initial. The dimensionless thickness Δ of the layer, defined as $|d \ln n/dx|^{-1}$, is equal to 10^{-3} in the first case and $5 \cdot 10^{-4}$ in the second. The magnetic field is expelled to the walls much less strongly than the plasma is ($H|_{x=1} = 5.7$ in the presence of a thermal force, $H|_{x=1} = 2.5$ without it). The reason for this is the important role of diffusion of the magnetic field on account of the high electron-scattering frequency in the Bohm case.

The efficiency is found to have the values $\eta_1 \approx 0.7$ and $\eta_2 = 0.6$. It is higher here than in the classical case because, at the same values of the dimensionless parameters ε, Δt, α, and γ, the ratio of the heating to the cooling time of the plasma is much smaller.

Figure 8 shows the profiles of the plasma density nT, the radiation density $n^2\sqrt{T}$, and the emission $\Delta = \int_0^x Q_{\mathrm{rad}}\, dx \Big/ \int_0^1$ at the time t^*. The plasma pressure is almost constant

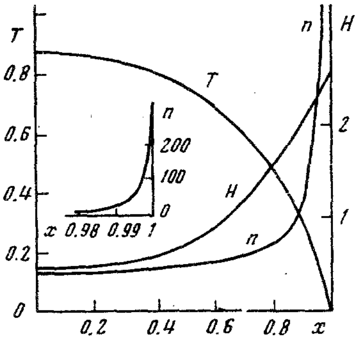

Fig. 7

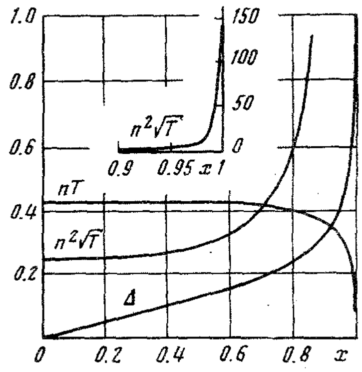

Fig. 8

TABLE 2

β_0	10^{-1}	1	10^2	10^3
T_{max}	0.698	0.706	0.79	0.75
n_m	0.58	0.58	0.54	0.43
$\tau_{1/2}$	1.6	1.7	1.7	2

TABLE 3

	$n_0 = 10^{18}$ cm^{-3}, $T_x = 10$ eV, $H_0 = 10^5$ G	
	with thermal force	without thermal force
L, cm	8	6.5
$\int Q_{rad}$, J·cm^{-2}·sec^{-1}	$2.9 \cdot 10^7$	$2.6 \cdot 10^7$
$Q_{rad}(\max)/Q_{rad}(\min)$	$6.5 \cdot 10^2$	$1.4 \cdot 10^4$
$q \mid_{x=L}$, J·cm^{-2}·sec^{-1}	$7.2 \cdot 10^7$	$2.7 \cdot 10^8$
δ,cm ($\delta = 1/\mid d\ln n/dx \mid_{max}$)	$8 \cdot 10^{-3}$	$3.3 \cdot 10^{-3}$
H_{max}/H_{min}	33.5	5.6
n_{max}, cm^{-3}	$6.8 \cdot 10^{19}$	$2.7 \cdot 10^{20}$
T_{max}, eV	10^4	10^4
$\tau_{1/2}$, sec	10^{-4}	10^{-4}

over the section, and magnetic pressure plays a role only at the wall itself. The graphs for the emission show that although the radiation density is high in the wall region, the region of the hot plasma makes the main contribution to the total emission.

Figures 9 and 10 show the density, magnetic field, and temperature as functions of the time in the center of the system and at the walls.

In Table 2 we give the maximal temperature in the center of the system, the density in the center of the time $t = t^*$, and the plasma cooling time $\tau_{1/2}$ for different values of β_0.

As can be seen from Table 3, in which we give the characteristics of a system with Bohm heat conduction for thermonuclear parameters of the plasma ($n_0 = 10^{18}$ cm^{-3}, $H_0 = 10^5$ G, $T_0 = 10^4$ eV), the total radiation losses may be an appreciable fraction of the loss due to heat conduction.

5. Scheme of Numerical Integration

For the numerical calculation, it was found to be important what variables were used to express the original equations. If the original system is expressed in Eulerian coordinates, their difference approximation on a spatial mesh with constant step leads to large errors in the determination of the gradients near the wall, which leads to the formation of large accelerations in the wall layer, the excitation of oscillations, and nonconservation of the mass. Introduction of the Lagrangian variable $dm = n(x)dx$ gives rise to a mesh that becomes denser in the region of high concentrations and therefore gives a more accurate approximation of the spatial derivatives. However, an attempt to satisfy the condition $\nabla(nT + H^2/\beta_0) = 0$ in the calculation led to a strong computational instability, which can be explained as follows. Under the condition of constancy of pressure over the whole volume, the concentration n is determined from the equation

$$nT + H^2 / \beta_0 = f(t) \tag{5.1}$$

and the velocity from the continuity equation. If (5.1) is differentiated with respect to the time, the values of dT/dt and dH/dt are substituted, and the condition $\int_0^1 n(x)\,dx = \text{const.}$, is used, one can determine the derivative df/dt. However, the errors of numerical differentiation and integration give an error of Δf at each time step. The maximal error in the determination of the concentration is near the wall, where $T \ll 1$, which leads ultimately

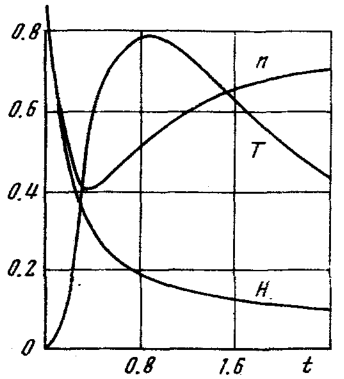

Fig. 9

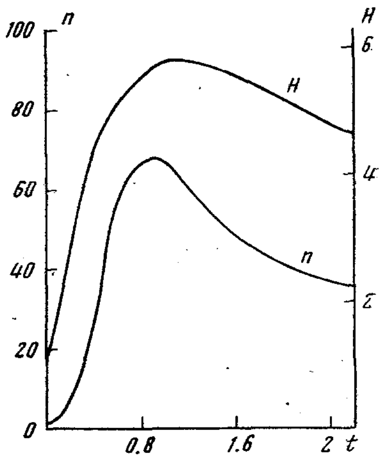

Fig. 10

to the appearance of the computing instability. Introduction of a small parameter ε into the equation of motion

$$\varepsilon \frac{dv}{dt} = -\frac{\partial}{\partial m}\left(nT + \frac{H^2}{\beta_0}\right)$$

enables one to establish a "normal" order of determination of the velocity and concentration. At the same time, the pressure over the whole volume is almost constant during the whole of the calculation time. From the system of different equations

$$\begin{aligned} &T_j^{m+1} - T_j^m + \frac{\tau}{3h}(nT)_j^m (v_{j+1}^m - v_{j-1}^m) = \frac{\tau}{h^2}\left\{\frac{(n_{j+1/2}^m)^2 T_{j+1/2}^m}{H_{j+1/2}^m}\delta_{2j+1/2}^m (T_{j+1}^{m+1} - T_j^{m+1}) - \right. \\ &\left. - \frac{(n_{j-1/2}^m)^2 T_{j-1/2}^m}{H_{j-1/2}^m}\delta_{2j-1/2}^m (T_j^{m+1} - T_{j-1}^{m+1})\right\} + \frac{4\tau}{3\beta_0^2 h}(H_{j+1}^m - H_{j-1}^m)\left\{\delta_{0j}^m H_j^m (H_{j+1}^m - H_{j-1}^m) + \right. \\ &\left. + \frac{\beta_0}{4}\delta_{1j}^m n_j^m (T_{j+1}^m - T_{j-1}^m)\right\} + \frac{2\tau}{3\beta_0 h^2}\left\{\delta_{1j+1/2}^m n_{j+1/2}^m T_{j+1/2}^m (H_{j+1}^m - H_j^m) - \right. \\ &\left. - \delta_{1j-1/2}^m n_{j-1/2}^m T_{j-1/2}^m (H_j^m - H_{j-1}^m)\right\} - \tau\left\{\alpha (n_j^m)^2 \sqrt{T_j^m}\frac{(T_j^m - \gamma)^2}{(T_j^m - \gamma)^2 + \gamma^2} - \right. \\ &\left. - \frac{1}{n_j^m}\frac{t^m}{(\Delta t)^2}\exp\left(-\frac{t^m}{\Delta t}\right)\frac{\exp(1 - x_j^2) - 1}{e - 1}\right\} \end{aligned} \tag{5.2}$$

$$v_j^{m+1} = v_j^m - \varepsilon^{-1}\frac{\tau}{h}\left(n_{j+1/2}^m T_{j+1/2}^m + \frac{(H_{j+1/2}^m)^2}{\beta_0} - n_{j-1/2}^m T_{j-1/2}^m - \frac{(H_{j-1/2}^m)^2}{\beta_0}\right) \tag{5.3}$$

$$x_j^{m+1} = x_j^m + \tau v_j^{m+1} \tag{5.4}$$

$$n_{j+1/2}^{m+1} = \frac{\Delta m_0}{x_{j+1}^{m+1} - x_j^{m+1}} \tag{5.5}$$

$$\begin{aligned} &\frac{H_j^{m+1}}{n_j^{m+1}} - \frac{H_j^m}{n_j^m} = \frac{4\tau}{\beta_0 h^2}\{\delta_{0j+1/2}^m H_{j+1/2}^m (H_{j+1}^{m+1} - H_j^{m+1}) - \\ &- \delta_{0j-1/2}^m H_{j-1/2}^m (H_j^{m+1} - H_{j-1}^{m+1})\} + \frac{\tau}{h^2}\{\delta_{1j+1/2}^m n_{j+1/2}^m (T_{j+1}^{m+1} - T_j^{m+1}) - \delta_{1j-1/2}^m n_{j-1/2}^m (T_j^{m+1} - T_{j-1}^{m+1})\} \end{aligned} \tag{5.6}$$

it can be seen that the field and, to a high accuracy, the mass are conserved in the considered volume. The difference equations of heat conduction and diffusion of the magnetic field were solved by the sweep method. The complete system of equations is conditionally stable. The time step was determined by

$$\tau = \alpha \min_j (\Delta x_j) / (1 + \max_j |v_j|)$$

where α was chosen from the condition of conservation of the integral $\int_0^1 H(x)\,dx$. For all the calculated variants, the integral was conserved within the range 0–0.3.

6. Main Results

Let us list the main results. We have demonstrated the possibility of containment of a dense plasma by walls. We have obtained acceptable solutions with a temperature drop from thermonuclear values to $T_x \sim 10^5$°K. We have established the characteristic features of nonmagnetic containment of a dense plasma: flow of plasma and the formation of a wall layer. We have shown that although these features do lead to additional energy losses from the plasma, the relative amount of these losses is of order unity. Therefore, the energy lifetime of the plasma remains of the same order as in the case of pure heat conduction, and it can be estimated by $\tau \sim L^2/\varkappa_\perp$ where $\varkappa_\perp$ is the thermal conductivity coefficient (classical or Bohm).

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